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Multi-process capability plot and fuzzy inference evaluation

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Abstract

Process capability indices C_p, C_{pk}, C_{pm} and C_{pp} fitting for nominal-the-best type quality characteristics, are effective tools to assess process capability since these indices can reflect a centering process capability and process yield adequately. The index C_{pp} introduced by Greenwith and Jahr-Schaffrath [Greenwith, M., Jahr-Schaffrath, B.L., 1995. A process incapability index. International Journal of Quality and Reliability Management 12, 58–71] provides additional and individual information concerning the process accuracy and the process precision. Although C_{pp} is useful to evaluate process capability for a single product in common situation, C_{pp} cannot be applied to evaluate the multi-process capability. Referring to Vännman and Deleryd's (C_{dr}, C_{dp})-plot, a fuzzy inference approach is proposed in our study to evaluate the multi-process capability based on distance values of a confidence box. This method takes the advantages of fuzzy systems such that a grade instead of sharp evaluation result can be obtained. An illustrated example of color STN display demonstrates that the presented method is effective for assessment of multi-process capability.

Keywords: Process capability indices; Nominal-the-best type quality characteristics; Confidence interval; Fuzzy inference

1. Introduction

Process capability indices (PCIs) are effective tools for the assessment of process capability indeed since the formulae of PCIs are easy to understand and straightforward to apply. Greenwith and Jahr-Schaffrath (1995) introduced a new index C_{pp}, which provides an uncontaminated separation between information concerning process accuracy and process precision. It has been widely used to provide numerical measures on whether a production is capable of producing items within the specification limits preset by the designer. The index C_{pp} can be defined as

$$C_{pp} = \left(\frac{\mu - T}{d/3}\right)^2 + \left(\frac{\sigma}{d/3}\right)^2, \tag{1}$$

where μ is the process mean, σ the process standard deviation, d the half the length of specification interval = (USL−LSL)/2, USL is the upper specification limit, LSL the lower specification limit, and T the target value. Now, let the inaccuracy index C_{dr} and imprecision index C_{dp} be defined as

$$C_{dr} = \frac{\mu - T}{d}, \tag{2a}$$

$$C_{dp} = \frac{\sigma}{d}. \tag{2b}$$

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Obviously, one can recognize that $C_{pp} = (3C_{dr})^2 + (3C_{dp})^2$ by comparing Eqs. (1) and (2). C_{pp} (including C_{dr} and C_{dp}) provides additional information concerning the process accuracy and the process precision. Index C_{pp} detects process inaccuracy and process imprecision by using indices C_{dr} and C_{dp}. Thus, C_{pp} is a deter choice for engineers measuring process potentials and performance. Under the assumption of normality, Chen (1998) first shows that the estimators of C_{pp} is a uniformly minimum variance unbiased estimator. It also obtains the rth moment and the probability density function of this estimator. Chen also takes into account the sampling errors and develop a simple procedure using the uniformly minimum variance unbiased estimator of C_{pp}, for practitioners to use in determining whether a process meets the capability requirement. The decisions made based on the proposed procedure are, of course, more reliable.

Although C_{pp} is useful to evaluate process capability for a single product in a common situation, C_{pp} cannot be applied to evaluate process capability for that of multi-process. Thus, we extend the applicability of the contour plot for processes with multiple characteristics. In addition, we also apply the method developed by Chen et al. (2003a, b) who introduced a process capability plot, called the MCPCA control chart, which is an adjustment of Vännman and Deleryd's (1999) (C_{dr}, C_{dp})-plot where $C_{dr} = (\mu - T)/d$ and $C_{dp} = \sigma/d$. Referring to (C_{dr}, C_{dp})-plot, a method to incorporate the fuzzy inference with process capability is used to assess multi-process. The concept of fuzzy sets was first proposed by Zadeh (1965). Now, fuzzy theorems have been applied in many fields such as automatic control, manufacturing system and decision-making (Gulley and Jang, 1996; Jang, 1993; Kacprzyk, 1997; Lin and Lin, 2001; Lin and Sheu, 1992; Mamdani, 1974; Takagi and Sugeno, 1985) in industry. In this paper, a fuzzy inference approach to multi-process capability is proposed. This fuzzy inference evaluation will consider C_{dr} and C_{dp} to formulate new indices as input and obtain a result value as output. In addition, illustrated example and evaluation procedure will be presented for ease of applications.

2. Relationship between process capability index and process yield

The index C_{dr} measures departure of process mean μ from the target value T. For centering process, $C_{dr} = 0$ it indicates that the process is completely on target, $C_{dr} = 1$ it indicates that $\mu = \text{USL}$, and $C_{dr} = -1$ it indicates that $\mu = \text{LSL}$. If $|C_{dr}| > 1$, μ falls outside the specification limits. The formula of C_{dr} transforms the original specifications form (LSL, T, USL) to $(-1, 0, 1)$. Table 1 displays various C_{dr} values and the corresponding values of μ.

While the index C_{dr} measures degree of process departure ratios, the index C_{dp} measures the magnitude of process variation. Because $C_{dr} = (\mu - T)/d$ and $C_{dp} = \sigma/d$, $C_{pp} = (3C_{dr})^2 + (3C_{dp})^2$ is achieved. For simplicity, let $C_{pp} = c$, then the relationship between index C_{pp} and process yield is

$$\%\text{Yield} = \Phi\left[\frac{1 + \sqrt{c/9 - (\sigma/d)^2}}{(\sigma/d)}\right] + \Phi\left[\frac{1 - \sqrt{c/9 - (\sigma/d)^2}}{(\sigma/d)}\right] - 1, \quad (3)$$

where Φ is the standard normal cumulative distribution function and $(\sigma/d) \leqslant (\sqrt{c}/3)$. As noted by Chen et al. (2001), process capabilities are categorized into five conditions. Table 2 presents the quality conditions and the corresponding C_{pm} and C_{pp} values. Let $h = 9/\sqrt{C_{pp}}(\sigma/d)$, the curves of the function of process yield versa C_{pp} with $h = 1, 2$ and 3 are shown in Fig. 1. One can recognize that a smaller value of C_{pp} corresponds to a high process yield as shown in Fig. 1. When $C_{dr} = 0$ (or $h = 3$), the relationship between index C_{pp} and process is $\%\text{Yield} = 2\Phi(3/\sqrt{C_{pp}}) - 1$ and when $|C_{dr}| \leqslant 1$, the relationship between index C_{pp} and process is $\%\text{Yield} = 2\Phi(3/\sqrt{C_{pp}}) - 1$. Practitioners are encouraged to pursue smaller values of C_{pp} (so as C_{dr} and C_{dp}) to ensure the satisfied process yields.

Table 1
C_{dr} values and the corresponding values of μ

C_{dr} values	Values of μ
$\lvert C_{dr}\rvert = 0$	$\mu = T$
$\lvert C_{dr}\rvert = 0.2$	$\mu = T \pm 0.2d$
$\lvert C_{dr}\rvert = 0.4$	$\mu = T \pm 0.4d$
$\lvert C_{dr}\rvert = 0.6$	$\mu = T \pm 0.6d$
$\lvert C_{dr}\rvert = 0.8$	$\mu = T \pm 0.8d$
$\lvert C_{dr}\rvert = 1$	$\mu = T \pm d$
$\lvert C_{dr}\rvert > 1$	Outside of limits

3. Process capability analysis plot

The index C_{pp} can directly be used to assess process capability when 100% inspection is applied. Instead of 100% inspection, acceptance sampling is most likely to be useful when the testing is destructive, or the testing cost is extremely high, etc. Generally, only estimated capability index $\hat{C}_{pp}$ by using a sample can be obtained in practice. Let $X_{i1}, X_{i2}, \ldots, X_{in}$, $i = 1, 2, \ldots, k$, are k sets of random samples of size n from each process (the samples size are selected the same for simplicity; however, it is not necessary). Each process has the same product specification and target value. The above calculation of average and variance are briefly summarized in Table 3.

Thus, the natural estimator of C_{ppi} can be written as the following:

$$\hat{C}_{ppi}\frac{(\bar{X}_i - T)^2}{D^2} + \frac{S_i^2}{D^2}, \quad i = 1, 2, \ldots, k, \tag{4}$$

Table 2
Classification of quality conditions

Quality condition	C_{pm}	C_{pp}
Inadequate	$C_{pm}<1$	$C_{pp}>1$
Capable	$1.00\leqslant C_{pm}<1.33$	$0.56<C_{pp}\leqslant 1.00$
Satisfactory	$1.33\leqslant C_{pm}<1.50$	$0.44<C_{pp}\leqslant 0.56$
Excellent	$1.50\leqslant C_{pm}<2.00$	$0.25<C_{pp}\leqslant 0.44$
Super	$C_{pm}\geqslant 2.00$	$C_{pp}\leqslant 0.25$

where $D = d/3$, $\bar{X}_i$ and S_i^2 are the sample mean and sample variance of process i with sample size n_i. The probability density function of $\hat{C}_{ppi}$ (see Chen, 1998) is

$$f_{\hat{C}_{pp}}(x) = \left(\frac{nD^2}{\sigma^2}\right)\sum_{j=0}^{\infty}P_j(\lambda)\int_0^{(nD^2/\sigma^2)x} f_K\left(\frac{nD^2}{\sigma^2}x - y\right) \times f_{Y_j}(y)\,\mathrm{d}y, \tag{5}$$

where $x>0$, Y_j is distributed as χ^2 with $(1+2j)$ degrees of freedom and

$$P_j(\lambda) = \frac{\mathrm{e}^{-(\lambda/2)}(\lambda/2)^j}{j!}. \tag{6}$$

Furthermore, the mean value and variance about $\hat{C}_{ppi}$ is

$$E = \left(\hat{C}_{ppi}\right)C_{pp},$$

$$\mathrm{Var}\left(\hat{C}_{ppi}\right) = \frac{2\sigma^4}{nD^4} + \frac{4(\mu - T)^2\sigma^2}{nD^4}.$$

Let $\hat{\mu}$ and $\hat{\sigma}$ be the unbiased estimators of μ and σ, then we have $\hat{\mu} = \bar{X}$, $\hat{\sigma} = c_4 \times S$ and $c_4 = \sqrt{2/(n-1)}\Gamma[n/2]/\Gamma[(n-1)/2]$. The factor c_4 is a function of the sample size n and c_4 approaches to unity when the sample size is large enough as shown in Fig. 2. Under normal assumption, $(n-1)[(c_4\hat{\sigma})/\sigma]^2$ is obeying the χ^2-distribution with $(n-1)$ degrees of freedom. Thus, the confidence intervals

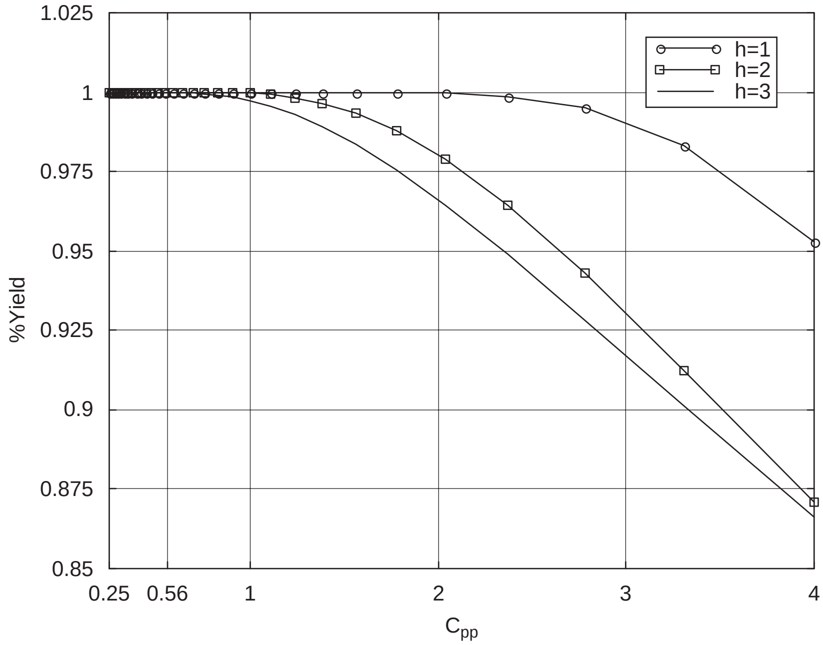

Fig. 1. The relationship between C_{pp} and process yield.

Table 3
Sample data mean and variance

Sample	Mean	Variance
$X_{11}, X_{12}, \ldots, X_{1n}$	$\bar{X}_1 = \left(\sum_{j=1}^{n} X_{1j}\right) \Big/ n$	$S_1^2 = \sum_{j=1}^{n} (X_{1j} - \bar{X}_1)^2 \Big/ (n-1)$
$X_{21}, X_{22}, \ldots, X_{2n}$	$\bar{X}_2 = \left(\sum_{j=1}^{n} X_{2j}\right) \Big/ n$	$S_2^2 = \sum_{j=1}^{n} (X_{2j} - \bar{X}_2)^2 \Big/ (n-1)$
$\vdots$	$\vdots$	$\vdots$
$X_{k1}, X_{k2}, \ldots, X_{kn}$	$\bar{X}_k = \left(\sum_{j=1}^{n} X_{kj}\right) \Big/ n$	$S_k^2 = \sum_{j=1}^{n} (X_{kj} - \bar{X}_k)^2 \Big/ (n-1)$

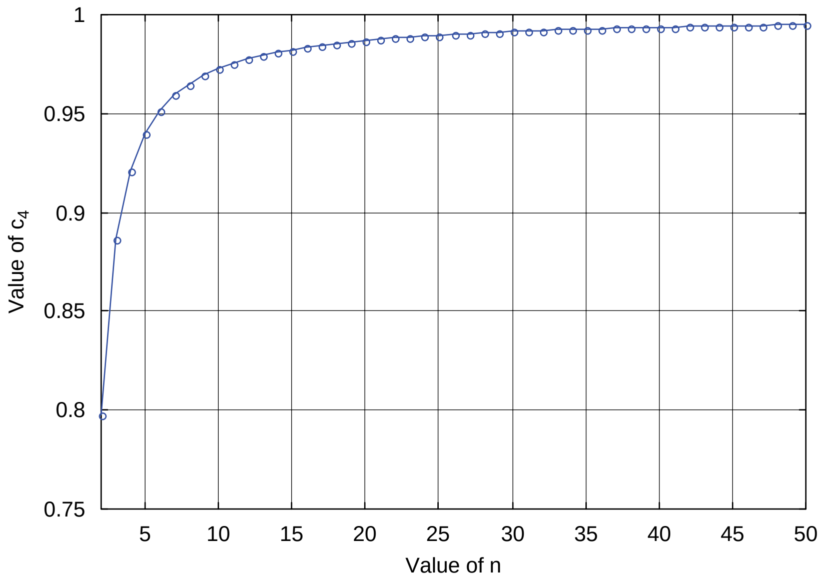

Fig. 2. Relationship between sample size n and factor c_4.

of μ and σ can be represented as

$$\mu : \left[\hat{\mu} - t_{\alpha/4,(n-1)} \times c_4 \times \frac{\hat{\sigma}}{\sqrt{n}}, \hat{\mu} + t_{\alpha/4,(n-1)} \times c_4 \times \frac{\hat{\sigma}}{\sqrt{n}}\right] = (X_l, X_u), \tag{6a}$$

$$\sigma : \left[\sqrt{\frac{(n-1) \times c_4^2 \times \hat{\sigma}^2}{\chi^2_{1-\alpha/4}(n-1)}}, \sqrt{\frac{(n-1) \times c_4^2 \times \hat{\sigma}^2}{\chi^2_{\alpha/4}(n-1)}}\right] = (Y_l, Y_u), \tag{6b}$$

where $t_{\alpha/4,(n-1)}$ is the upper quartile of t distribution with $(n-1)$ degrees of freedom; $\chi^2_{1-\alpha/4}(n-1)$ and $\chi^2_{\alpha/4}(n-1)$ are the upper percentile of χ^2-distribution with $(n-1)$ degrees of freedom. The joint confidence intervals of μ and σ are used to formulate a confidence region and applied to reveal the process capability. The four coordinates of this confidence region are thus represented as

Upper-right coordinate:

$$\left(\hat{\mu} + t_{\alpha/4,(n-1)} \times c_4 \times \frac{\hat{\sigma}}{\sqrt{n}},\ \sqrt{\frac{(n-1) \times c_4^2 \times \hat{\sigma}^2}{\chi^2_{\alpha/4}(n-1)}}\right), \tag{7a}$$

Bottom-right coordinate:

$$\left(\hat{\mu} + t_{\alpha/4,(n-1)} \times c_4 \times \frac{\hat{\sigma}}{\sqrt{n}},\ \sqrt{\frac{(n-1) \times c_4^2 \times \hat{\sigma}^2}{\chi^2_{1-\alpha/4}(n-1)}}\right), \tag{7b}$$

Upper-left coordinate:

$$\left(\hat{\mu} - t_{\alpha/4,(n-1)} \times c_4 \times \frac{\hat{\sigma}}{\sqrt{n}},\ \sqrt{\frac{(n-1) \times c_4^2 \times \hat{\sigma}^2}{\chi^2_{\alpha/4}(n-1)}}\right), \tag{7c}$$

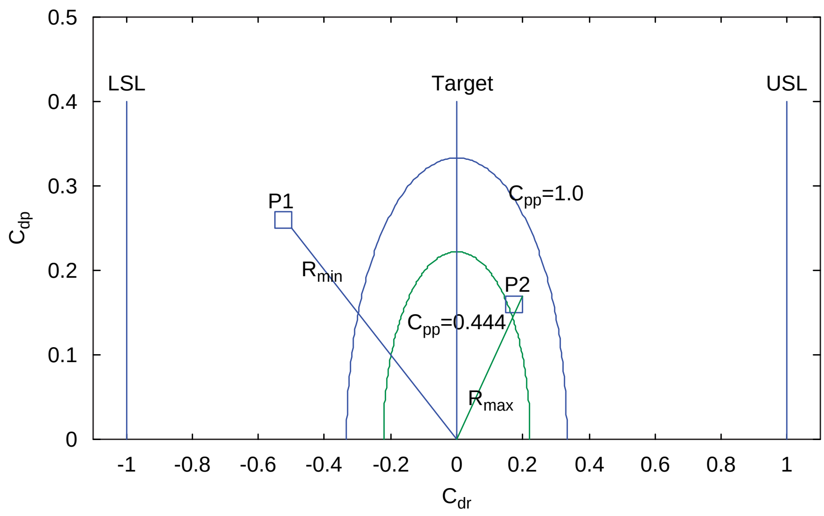

Fig. 3. Multi-process capability analysis plot.

Bottom-left coordinate:

$$\left(\hat{\mu} - t_{\alpha/4,(n-1)} \times c_4 \times \frac{\hat{\sigma}}{\sqrt{n}}, \sqrt{\frac{(n-1) \times c_4^2 \times \hat{\sigma}^2}{\chi^2_{1-\alpha/4}(n-1)}}\right). \tag{7d}$$

Referring to Vännman and Deleryd's (C_{dr}, C_{dp})-plot, we propose a new method to assess the process capabilities of multi-process. Since $C_{dr} = (\mu - T)/d$ and $C_{dp} = \sigma/d$, the confidence region (described in Eq. (7)) in the capability plot of $C_{dr}-C_{dp}$ should be changed as

Upper-right coordinate:

$$((X_u - T)/d, Y_u/d) = (X_{ru}, Y_{ru}), \tag{8a}$$

Bottom-right coordinate:

$$((X_u - T)/d, Y_u/d) = (X_{ru}, Y_{rl}), \tag{8b}$$

Upper-left coordinate:

$$((X_l - T)/d, Y_u/d) = (X_{rl}, Y_{ru}), \tag{8c}$$

Bottom-left coordinate:

$$((X_l - T)/d, Y_l/d) = (X_{rl}, Y_{rl}), \tag{8d}$$

where X_u, X_l, Y_u and Y_l are described in Eq. (6). And the maximum estimated index $(\hat{C}_{pp})_{max}$ could be calculated as

$$(\hat{C}_{pp})_{max} = \max[(3X_{ru})^2 + (3Y_{ru})^2, (3X_{rl})^2 + (3Y_{ru})^2]. \tag{9}$$

To reduce the influence of sampling errors, we shall use values of mentioned confidence box described in Eq. (8) to afford a more reliable assessment. In the plot of $C_{dr}-C_{dp}$, there are two process capability values for P1 and P2 as seen in Fig. 3, one can recognize that the process capability is adequate if the confidence box is inside of the line $C_{pp} = 1$ (process P2) and process capability is inadequate if the confidence box is outside of the line $C_{pp} = 1$ (process P1). The larger distance of this confidence box to the coordinate original point (0, 0), the worse process capability. Let the nearest distance from the coordinate original point to each confidence box be R_{min} in the capability plot (Fig. 3) and the most far distance from the coordinate original point to each confidence box be R_{max}, then R_{min} and R_{max} can be defined referring to Eq. (8) as

$$R_{min} = \min\left(\sqrt{X_{ru}^2 + Y_{rl}^2}, \sqrt{X_{rl}^2 + Y_{rl}^2}, Y_{rl}\right), \tag{10}$$

$$R_{max} = \max\left(\sqrt{X_{ru}^2 + Y_{ru}^2}, \sqrt{X_{rl}^2 + Y_{ru}^2}\right). \tag{11}$$

In our study, R_{min} and R_{max} are used to represent the process capability for each model and a reliable assessment is achieved by using the magnitudes of R_{min} and R_{max}.

4. Fuzzy inference method for process capability

In this section, a fuzzy inference approach is proposed to assess the process capability for multi-process. A process capability analysis plot as stated in Section 3, R_{min} and R_{max} are used to assess the process capability. The larger values of R_{min} and R_{max} for each model, the worse process capability.

Let $[R_{\min i}, R_{\max i}]$ and $[R_{\min j}, R_{\max j}]$ be the nearest and the most far distances in the plot of $C_{dr}-C_{dp}$ for the process i and j, respectively. Consider $[R_{\min i}, R_{\max i}]$ and $[R_{\min j}, R_{\max j}]$ as set of two lines in same axis, then the comparison of two processes can be represented by statistical method as

(1) If $[R_{\min i}, R_{\max i}] \cap [R_{\min j}, R_{\max j}] \neq \phi$, then concluded that equal process capability for process i and j.
(2) If $R_{\min i} > R_{\max j}$, then concluded that the process capability for process i is inferior to that of process j.
(3) If $R_{\max i} < R_{\min j}$, then concluded that the process capability for process i is superior to that of process j.

Nevertheless, it is rather ambiguous in rule one. In this rule, the process capability is concluded to be equal irrespective of whether the intersection is small or large. For instance:

Case A: When $[R_{\min i}, R_{\max i}] = [0.1, 0.6]$ and $[R_{\min j}, R_{\max j}] = [0.5, 0.9]$ then $[R_{\min i}, R_{\max i}] \cap [R_{\min j}, R_{\max j}] = [0.5, 0.6]$.

Case B: When $[R_{\min i}, R_{\max i}] = [0.2, 0.8]$ and $[R_{\min j}, R_{\max j}] = [0.3, 0.9]$ then $[R_{\min i}, R_{\max i}] \cap [R_{\min j}, R_{\max j}] = [0.3, 0.8]$.

One can recognize *Case B* is superior to *Case A* for the possibility of equal process capability. To deal with the ambiguous problem in judging process capability, a method to incorporate the fuzzy inference with a process capability index is proposed. Thus, distinguishing the equal grade of capability (for process i and j in rule one) in different intersection is achieved. An approximating rule-based reasoning approach is used for quantitative analysis. In our study, the process i is said to be superior to process j when the value of inference result is positive. The larger the result value, the more capable of process i is compared to process j. Oppositely, the negative value of result implies that the process i is inferior to process j. In other words, the process i is said to be completely better-quality than that of process j when the value of inference result is equal to 1; the process i is said to be completely same-quality to that of process j when the value of inference result is equal to 0 and the process i is said to be completely worse-quality than that of process j when the value of inference result is equal to -1. The result value of inference within $\{0, 1\}$ or $\{-1, 0\}$ is used to represent the different grade of capability. The h processes are tested two at a time, there are ${}_hC_2 = h(h-1)/2$ possible paired comparisons. Let the indices δ and γ be defined as

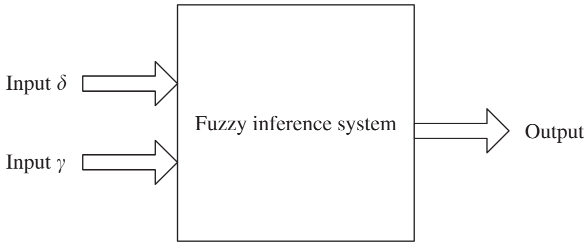

Fig. 4. Structure of fuzzy inference system.

$$\delta = \frac{R_{\min i} - R_{\max j}}{\max(R_{\max i}, R_{\max j})}, \tag{12}$$

$$\gamma = \frac{R_{\max i} - R_{\min j}}{\max(R_{\max i}, R_{\max j})}. \tag{13}$$

Then the fuzzy inference systems are composed of two inputs and one output as shown in Fig. 4.

A constructive theory of fuzzy sets is proposed by Zadeh (1965). He replaced the characteristic function in the conventional set theory, which takes on the value 0 or 1, by the so-called membership function. Generally, the procedure of fuzzy analysis consists of four steps: definition of input/output fuzzy variables, fuzzy rules, fuzzy inference and defuzzification (Jang, 1993; Kacprzyk, 1997; Lin and Lin, 2001; Lin and Sheu, 1992; Mamdani, 1974; Takagi and Sugeno, 1985; Vännman and Deleryd, 1999; Yager and Filev, 1994).

(1) *Definition of input/output fuzzy variables*: The membership functions (MFs) of input/output variables are defined by linguistic variables. There are four kinds of MFs for representing fuzzification: triangular, trapezoid, Gaussian and sigmoid type. In our study, we adopt the triangular and trapezoid types as MFs for the sake of simplicity and easy to describe the asymmetric property. The triangular MF is specified by three parameters $\{a, b, c\}$ which determine the three corners of triangle. If this function is denoted as trimf(x;a,b,c) then

$$\text{trimf}(x; a, b, c) = \begin{cases} 0 & x < a, \\ \frac{x-a}{b-a} & a \leqslant x \leqslant b, \\ \frac{c-x}{c-b} & b \leqslant x \leqslant c, \\ 0 & x > c. \end{cases} \tag{14}$$

Furthermore, the trapezoid MF is denoted trapmf(x;a,b,c,d) which is specified by four

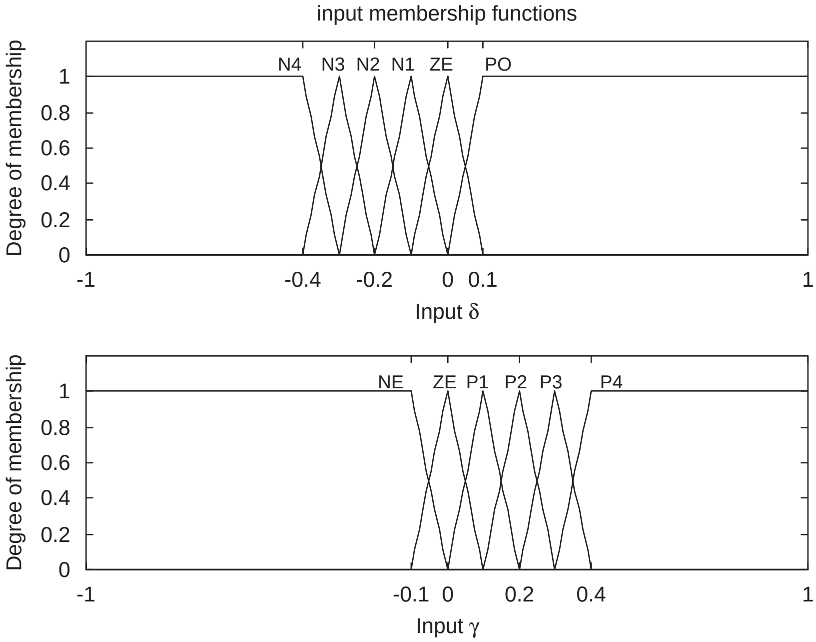

Fig. 5. Membership functions of input variables.

parameters $\{a,b,c,d\}$, and then we have

$$\mathrm{trapmf}(x;a,b,c,d)=\begin{cases}0 & x<a,\\ \frac{x-a}{b-a} & a\leqslant x\leqslant b,\\ 1 & b\leqslant x\leqslant c,\\ \frac{d-x}{d-c} & c\leqslant x\leqslant d,\\ 0 & x>d.\end{cases} \quad (15)$$

The universe of input variables is defined in $\{-1, 1\}$ as shown in Fig. 5. MFs of input δ are defined as trapmf(x;−1,−1,−0.4,−0.3), trimf (x;−0.4,−0.3,−0.2), trimf(x;−0.3,−0.2,−0.1), trimf(x;−0.2,−0.1,0), trimf(x;−0.1,0,0.1) and trapmf(x;0,0.1,1,1) for representing N4 (negative), N3, N2, N1, ZE (zero) and PO (positive), respectively. Also, input γ are defined as trapmf(x;−1,−1,−0.1,0), trimf(x;−0.1,0,0.1), trimf (x;0,0.1,0.2), trimf(x;0.1,0.2,0.3), trimf(x;0.2, 0.3,0.4) and trapmf(x;0.3,0.4,1,1) for representing NE (negative), ZE, P1 (positive), P2, P3 and P4, respectively. In addition, the output variables are composed of seven triangular MFs for representing L3 (inferior), L2, L1, EQ, S1 (superior), S2 and S3 as shown in Fig. 6.

(2) *Fuzzy rules*: Fuzzy rules are important to successful inference result (Gulley and Jang, 1996; Jang, 1993; Kacprzyk, 1997; Lin and Lin, 2001; Lin and Sheu, 1992; Mamdani, 1974; Takagi and Sugeno, 1985; Vännman and Deleryd, 1999; Yager and Filev, 1994). A rule base represents the experience and knowledge of experts. The fuzzy rules are similar to the intuitional thinking of a human. In general, there are no systematic tools for forming the fuzzy rules. Different sources of knowledge, resulting in formulation of alternative rules, can be considered. A fuzzy inference system, composed of two inputs and one output, could employ this kind of fuzzy rule as

If x_1 is A_{i1} and x_2 is A_{i2} then y is B_i (for $i=1-n$),

where x_1, x_2 and y are fuzzy system input and output variables; A_{i1}, A_{i2} and B_i are fuzzy subsets of their linguistic variables. In this study, the fuzzy inference system is applied to assess the process capability by using the confidence interval values of $R_{\min}$ and $R_{\max}$ (defined in Eq. (8)). Thirty-three *if–then* rules are employed in our study. They are:

Rule 1: if (δ is PO) and (γ is P4) then (result is L3).

Rule 2: if (δ is PO) and (γ is P3) then (result is L3).

⋮

Rule 33: if (δ is N4) and (γ is NE) then (result is S3).

The tabulated fuzzy rules are listed in Table 4. Note here, the fuzzy rules indicated in Table 4, if

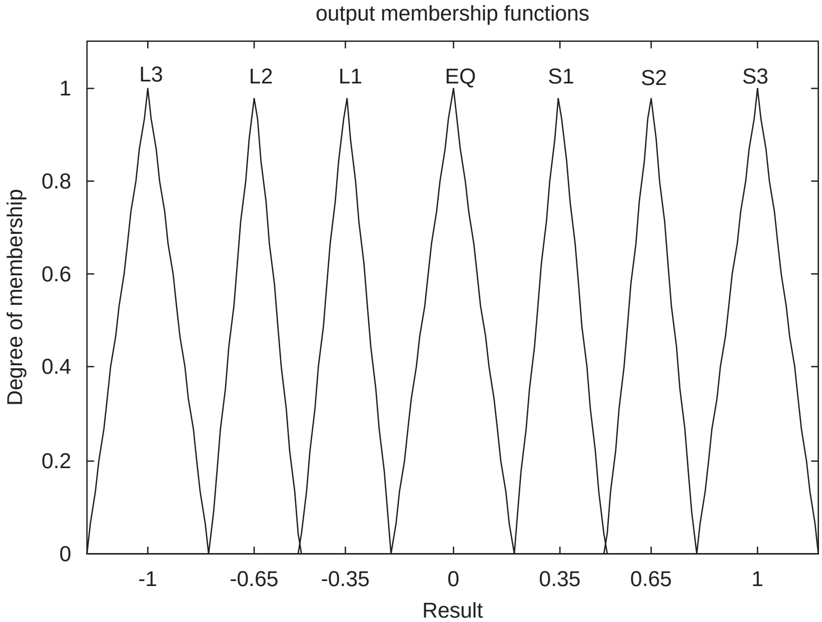

Fig. 6. Membership functions of output variable.

(δ is PO) and (γ is ZE) or (γ is NE) in addition to if (δ is ZE) *and* (γ is NE), are never happened since the definition of two input variables always exists $\gamma \geqslant \delta$.

(3) *Fuzzy inference*: Fuzzy inference is an inference procedure to derive conclusion based on a set of *if–then* rules. In this paper, the Mamdani inference method (Kacprzyk, 1997) that employs the maximum–minimum product composition to operate fuzzy *if–then* rules is adopted (another inference method see Mamdani, 1974). Let the rule be: if $x_1 = A_1$ and $x_2 = A_2$ then $y = B$, then the result of inference can obtain a fuzzy set with MF of B_i' as

$$\mu_{B_i'}(y) = \max_X \left\{ \min[\mu_{A_{i1}'}(x_1), \mu_{A_{i2}'}(x_2), \mu_{R_i}(x_1, x_2, y)] \right\}, \quad (16)$$

where

$$\mu_{R_i}(x_1, x_2, y) = \min[\mu_{A_{i1}}, \mu_{A_{i2}}, \mu_{B_i}(y)].$$

(4) *Defuzzification*: The fuzzy sets of B_i' are obtained by Step (3), then the defuzzification is used to find a crisp value $y^* \in Y$ which represents the fuzzy sets. The frequently used defuzzification methods have: weight, area and height method in Yager and Filev (1994). The weight defuzzification method is used in our study, and then we have

$$y* = \frac{\int_Y yB(y)\,\mathrm{d}y}{\int_Y B(y)\,\mathrm{d}y}. \quad (17)$$

Result of fuzzy inference, performed by a *Matlab Logic Fuzzy Toolbox* (Zadeh, 1965), is then used to represent the process capability for each model.

Table 4
The tabulated fuzzy rules

γ	P4	P3	P2	P1	ZE	NE
δ						
PO	L3	L3	L3	L3	—	—
ZE	L2	L2	L2	L1	EQ	—
N1	L2	L2	L1	EQ	S1	S3
N2	L1	L1	EQ	S1	S2	S3
N3	L1	EQ	S1	S2	S2	S3
N4	EQ	S1	S1	S2	S3	S3

5. Procedure of fuzzy evaluation and illustrated example

Below, an example is taken to explain the proposed procedure in detail. To illustrate how the testing procedure may be applied to the practical data colleted from the factories, the following case on color STN displays product was taken from a manufacturing industry located on middle Taiwan. Color STN displays are created by adding color filter to traditional monochrome STN displays. In color STN displays, each pixel is divided into red, green, and blue sub-pixels. To control the light through the color filter, different colors are made by combination of these primary colors. The thickness of membrane, which plays as an important quality characteristic in our study, is measured for each pixel after finishing post-baking process. The specification limits are set to $1200 \pm 50\,\mathrm{nm}$ ($1\,\mathrm{nm} = 10^{-9}\,\mathrm{m}$), that is, the upper/lower specification limits are set to USL = 1250, LSL = 1150, and the target value is set to $T = 1200$. In practice, if the thickness of membrane for color STN does not fall within the tolerance (LSL, USL), the problem of chromatic aberration for color STN displays will be happened. Table 5 presents the concise information of four models denoted with MOD1, MOD2, MOD3 and MOD4. The fuzzy evaluation procedure is stated as follows:

Step 1: Determine the sample size $n = 60$ for all manufacturing processes, then the values of mean and standard deviation are calculated as indicated in Table 5. Also, the significant level is given 0.05.

Step 2: Compute the values of c_4 and $\hat{\sigma}$. Also, calculate four coordinates values of each confidence box described in Eq. (8).

Table 5
Process capability value for four processes

Process	$\hat{\mu} = \bar{X}_i$	S_i	$\hat{\sigma}$	$R_{\min\ i}$	$R_{\max\ i}$	$(\hat{C}_{pp})_{\max i}$
MOD1	1203	10.0	0.9577	0.1657	0.2782	0.6966
MOD2	1201	10.1	10.0573	0.1720	0.2661	0.6373
MOD3	1200	11.1	11.0531	0.1954	0.2866	0.7394
MOD4	1197	10.6	10.5552	0.1756	0.2934	0.7747

Step 3: In the process capability analysis plot, compute the nearest and the most far distances ($R_{\min}$ and $R_{\max}$) in Eqs. (8) and (9) for each process.

Step 4: Compute the indices δ and γ for two-process pairs (there are six pairs) and thus to obtain the fuzzy evaluation results through the proposed fuzzy inference system. The above calculations are performed through developed *Matlab* program. As indicated in Table 6, one can recognize the model of MOD2 is the best one among these four processes since all values of inference result are positive for competing pairs (MOD2 to MOD*j*, for $j = 1$, 3 and 4).

6. Conclusion

Process capability indices like C_{pm} and C_{pp} are proved to reflect the centering process capability and process yield adequately and are used to achieve a numerical measure on whether a production is capable of producing items within the specification limits. The index C_{pp} is a simple transformation from the index C_{pm}, and provides individual information concerning the process accuracy and process precision. Referring to Vännman and Deleryd's (C_{dr}, C_{dp})-plot, a method to incorporate fuzzy inference with process capability is adopted in this paper to evaluate the capability of competing processes based on distance values in the process capability analysis plot. This fuzzy evaluation method considers the indices δ and γ (both are relative to the above confidence box) as inputs and obtain a result value as output. Results of inference are used to represent the grade of process capability for each model. The presented method affords a more reliable assessment result and possesses the advantages of fuzzy systems such that a grade instead of sharp evaluation result can be obtained. An illustrated example of color STN display is

Table 6
Fuzzy inference results

Pairs (*i*–*j*)	$[R_{\min\ i}, R_{\max\ i}]$	$[R_{\min\ j}, R_{\max\ j}]$	δ	γ	Result
MOD 1–2	[0.1657, 0.2782]	[0.1720, 0.2661]	−0.3610	0.3816	−0.0493
MOD 1–3	[0.1657, 0.2782]	[0.1954, 0.2866]	−0.4220	0.2891	0.3498
MOD 1–4	[0.1657, 0.2782]	[0.1756, 0.2934]	−0.4353	0.3496	0.1505
MOD 2–3	[0.1720, 0.2661]	[0.1954, 0.2866]	−0.3997	0.2469	0.3468
MOD 2–4	[0.1720, 0.2661]	[0.1756, 0.2934]	−0.4136	0.3084	0.2880
MOD 3–4	[0.1954, 0.2866]	[0.1756, 0.2934]	−0.3341	0.3783	0.0860

Note: Result value of "MOD*j* to MOD*i*" = −"MOD*i* to MOD*j*".

employed to demonstrate that the presented method is effective and thus supports its feasibility for assessment the capability of multi-process.

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